**Lecture** **5.**

**One-Sided Limits. Infinite limits. Monotonic Functions.**

**Definition . (**theright-hand limit of ).

We write **** to indicate that as approaches from the right, approaches , i.e. if , then we write conventionally The number  is called the *limit from the right* of the function.

**Definition . (**theleft-hand limit of ).

We write **** to indicate that as approaches from the left, approaches i.e. if , then we write conventionally The number  is called the *limit from the left* of the function.

**Definition.** The right-hand limit and left-hand limit are called *one-sided limits.*

*For a full limit to exist, both one-sided limits have to exist and they have to be equal.*

**First remarkable limit :**

 (4)

The formula (4) is frequently used when solving the following examples.

**Example 1:** 

**Example 2:**  

**Theorem (**The squeezing theorem). If *f, g*, and *h* be functions satisfying

Let *p>0*. Suppose that, for all *x* such that

If , then

**Monotonic Functions**

**Definition.** The function *f* is increasing on an interval I, if whenever are in I, and , or decreasing on an interval I, if whenever are in I, and

In either of these two case, f is strictly monotonic on I.

The function *f* is nondecreasing on an interval I, if whenever are in I, and , or nonincreasing on an interval I, if whenever are in I, and

A functions that satisfies any of these conditions is called *monotonic*.

**Definition.**

We say that *f* is bounded on a set S, if there is a constant such that

for all x in S.